MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

TRIMESTER 3, 2015/2016

FINAL EXAMINATION

EMT2046 – ENGINEERING MATHEMATICS IV

(CE, EE, LE, MCE, NE, OPE, TE)

30 MAY 2016

2:30 PM – 4:30 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **5 pages** (including cover page) with **four questions** only.

2. Attempt **ALL questions**. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Let X be a continuous random variable with probability density function

$$f_{XY}(x,y)=\begin{cases}\frac{1}{5}\left(12x^2+8xy^3\right), & 0<x\le 1,\ 0<y<1,\\ 0, & \text{otherwise.}\end{cases}$$

Find $E(XY)$.

[10 marks]

(b) Consider a Simple Random Walk with state space $S=\{1, 2, 3, 4\}$ and one-step transition probability matrix, P as follows:

$$P=\begin{bmatrix}0.2 & 0.8 & 0 & 0\\ 0 & 0 & 0.4 & 0.6\\ 1 & 0 & 0 & 0\\ 0.7 & 0 & 0.3 & 0\end{bmatrix}$$

(i) Draw the state transition diagram. [3.5 marks]

(ii) What is the period of the state 4? [2 marks]

(iii) Is this Markov chain ergodic? [2 marks]

(c) Consider a transition matrix with state space $\{0, 1\}$ as follows:

$$\begin{bmatrix}0.6 & 0.4\\ 0.3 & 0.7\end{bmatrix}$$

What is the limiting state (stationary) probability vector $\begin{bmatrix}\pi_0 & \pi_1\end{bmatrix}$?

[7.5 marks]

Continued...

Question 2

(a) Consider the following Runge Kutta method of order three for approximating the solutions of the differential equation $y' = f(x, y)$.

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 4k_2 + k_3)$$
$$k_1 = h f(x_i, y_i)$$
$$k_2 = h f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$
$$k_3 = h f(x_i + h, y_i - k_1 + 2k_2)$$

Use the above formula with step size $h = 0.2$ to approximate $y(0.4)$ when

$$y' = y - x^2, \quad y(0) = 2.$$

[13 marks]

(b) Find the first two iterations of the Gauss-Seidel method with $\mathbf{x}^{(0)} = \mathbf{0}$ in approximating the solutions for the following system of linear equations.

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 6 \\ 2x_1 + 6x_2 + x_3 &= 6 \\ -x_2 + 2x_3 &= 6 \end{aligned}$$

[12 marks]

Continued...

Question 3

(a) A toy manufacturer produces three products every day. The two main raw materials used for producing these products are cloths and plastics. Table 3A shows the cost of raw materials per product.

Raw Materials	Toy A	Toy B	Toy C
Cloths	RM1.00	RM1.50	RM1.20
Plastics	RM2.50	RM3.00	RM2.80

Table 3A

The daily costs spent on cloths and plastics are limited to RM2200 and RM5000, respectively. At least 50 Toy A is produced for donation to charity. Formulate a linear programming model to determine the number of products Toy A, Toy B and Toy C that should be produced daily in order to minimize the production cost. (Do not solve the problem)

[8 marks]

(b) Table 3B shows the initial simplex tableau for a maximization linear programming problem.

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	−1	−2	−1	0	0	0
s_1	0	1	−1	2	1	0	1
s_2	0	2	1	−1	0	1	5

Table 3B

(i) Formulate the original linear programming model.

[4 marks]

(ii) Solve the linear programming problem using simplex method.

[13 marks]

Continued...

Question 4

Figure 4 shows a graph representing the possible routes connecting 8 cities and the distances between these cities.

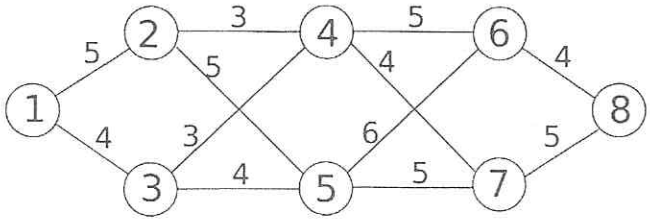

Figure 4

(a) Determine the shortest route from City 1 to City 8 using dynamic programming.

[14 marks]

(b) Apply Prim's Algorithm to the graph to find the minimal spanning tree.

[11 marks]

End of paper.